

Effect of Soil Layer Permeability on Seepage Beneath Hydraulic Structures: An Explainable Machine Learning Approach

Dilshad A.H. Alhadrawi

Department of Building and Construction Techniques, College of Technical Engineering, The Islamic University,
Najaf, Iraq
dilshadeng61@iunajaf.edu.iq

Abstract

Alluvial foundations of weirs and barrages are stratified, yet classical design methods and existing machine-learning (ML) surrogates treat the foundation as a single homogeneous layer. This study quantifies how the permeability of soil layers controls the seepage discharge (q), uplift force (U) and exit gradient (i_e) beneath a floor with upstream and downstream cutoffs, and proposes a physics-guided deep-ensemble (PG-DE) surrogate. A finite-volume solver, verified against exact conformal-mapping solutions (deviation $\leq 0.31\%$), generated 16,384 two-layer cases with permeability ratios $k_1/k_2 = 0.01-100$. PG-DE learns similarity-normalised responses from dimensionless (Buckingham- Π) groups. On unseen test cases it achieved $R^2 = 0.99992$, 0.99996 and 0.99991 for q , U and i_e , with mean absolute percentage errors $\leq 0.43\%$, whereas seven conventional learners trained on raw variables reached at most $R^2 = 0.876$, 0.969 and 0.976 . On a public MODFLOW exit-gradient database, the same physics-guided strategy raised R^2 from 0.710 to 0.902 under the original cross-validation protocol. Relative to homogeneous soil, the layer contrast changed q by factors of $0.06-82$ and i_e by up to 5.3 ; a less permeable surface layer sharply raises i_e , while uplift is most sensitive when the downstream cutoff tip reaches the layer interface. The framework offers an accurate and explainable tool for preliminary design on stratified foundations.

Keywords: seepage; soil layer; permeability; hydraulic structures; exit gradient; explainable machine learning

1. Introduction

Seepage beneath weirs, barrages and regulators founded on pervious soil causes water loss, uplift on the floor and, where the exit gradient approaches the critical value, piping. Practical design still relies on Lane's weighted creep [1], Khosla's method of independent variables [2] and conformal-mapping solutions [3], all of which assume a homogeneous isotropic foundation. Natural alluvial deposits, however, are stratified, and the permeability of adjacent layers may differ by several orders of magnitude.

Numerical studies confirm that layering governs seepage behaviour. Jamel [4] simulated 729 two-layer cases and found that discharge and exit gradient increase with the upper-layer permeability, whereas Mansour et al. [5] reported that the trend reverses depending on whether the upper layer is more or less permeable than the lower one. Experiments and simulations further show that the response changes abruptly when a cutoff penetrates the layer interface [6], [7], and that anisotropy [8] and random heterogeneity [9] strongly modify exit gradients. These partly conflicting findings indicate that the layer effect depends on interacting geometric and hydraulic variables that charts cannot capture.

Machine learning has therefore been adopted as a fast surrogate for numerical seepage models: feed-forward networks [10], ensemble networks [11], CatBoost [12], XGBoost with SHAP [13], gene-expression programming [14], [15] and optimised ANNs [16], [17] have reported coefficients of

determination between 0.88 and 0.9997, and CatBoost was recently applied to heterogeneous earthfill dams [18]. Three gaps remain. First, no surrogate uses the permeabilities and thickness of foundation layers as inputs; the most general model treats permeability only as anisotropy [13]. Second, models are trained on raw dimensional variables, ignoring the similarity laws of seepage, which inflates data requirements and degrades accuracy when responses span several orders of magnitude. Third, the data are rarely public, preventing independent comparison.

This study addresses these gaps by (i) developing and verifying a finite-volume seepage solver and a 16,384-case two-layer database; (ii) proposing a physics-guided deep ensemble that embeds dimensional similarity; (iii) benchmarking the strategy on a public database under the original authors' protocol; and (iv) explaining, through SHAP, how layer permeability controls q , U and i_e .

2. Materials and Methods

2.1. Problem definition and numerical model

Fig. 1 shows the analysed configuration: an impervious floor of length b with upstream and downstream cutoffs of depths d_1 and d_2 , founded on two horizontal layers of thickness t_1 and $T - t_1$ and permeabilities k_1 and k_2 , under a head difference H . Steady confined flow obeys $\partial/\partial x(k \partial h/\partial x) + \partial/\partial y(k \partial h/\partial y) = 0$. The equation was solved by a cell-centred finite-volume scheme on graded grids ($\approx 45,000$ cells), using harmonic-mean face permeabilities across the interface and zero-flux faces along cutoffs. The outputs are q (integrated exit flux), $U = \gamma_w \int h \, dx$ along the floor, and i_e , the vertical gradient averaged over $0.02T$ downstream of the downstream cutoff. The solver reproduced the exact conformal-mapping solutions [3] for a single sheet pile and for a flat floor within 0.06–0.31% on the production grid ($\leq 0.02\%$ on refined grids), with a mass-balance error below 4×10^{-10} .

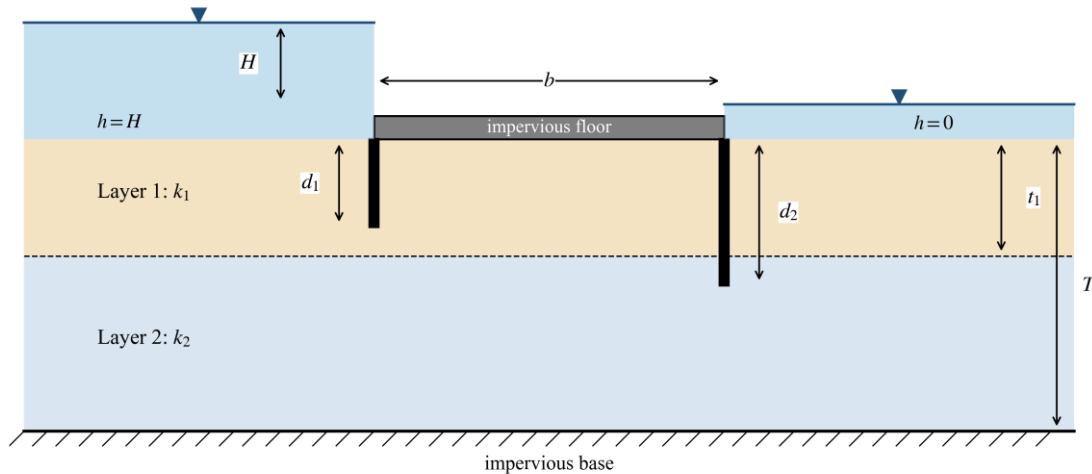


Fig. 1. Definition sketch of the hydraulic structure on a two-layer foundation.

2.2. Databases

The two-layer database comprises 16,384 cases drawn by scrambled Sobol sampling over the ranges in Table 1, which cover upper layers 100 times less to 100 times more permeable than the lower layer and cutoff tips above or below the interface. The public benchmark is the MODFLOW database of Rath et al. [19], which contains three independent samples of 1,999 single-layer simulations (Models5–7) with anisotropy, geometry and heads as inputs and the exit gradient as output [13]. To avoid information leakage, Models6 was used only for hyper-parameter tuning, and Models5 and Models7 only for evaluation.

Table 1. Input ranges of the two-layer database (16,384 cases).

Variable	Symbol	Range	Unit
Head difference	H	2–10	m
Floor length	b	10–50	m
Pervious depth	T	10–40	m
Upstream cutoff depth	d_1	0–0.7 T	m
Downstream cutoff depth	d_2	0.05 T –0.7 T	m
Upper-layer thickness	t_1	0.1 T –0.9 T	m
Lower-layer permeability	k_2	10^{-6} – 10^{-3}	m/s
Permeability ratio	k_1/k_2	10^{-2} – 10^2	–

2.3. Physics-guided learning framework

Because the governing equation is linear in h and homogeneous in k , dimensional analysis [20] gives $q = k_h H f_1(\Pi)$, $U = \gamma_w H b f_2(\Pi)$ and $i_e = (H/T) f_3(\Pi)$, where $\Pi = \{b/T, d_1/T, d_2/T, t_1/T, \log k_1/k_2\}$ and $k_h = (k_1 t_1 + k_2(T - t_1))/T$. Π was augmented with three physically motivated groups: the cutoff-tip offsets $(d_{1,2} - t_1)/T$, the logarithms of the parallel and series equivalent permeabilities, and the normalised transmissivity of the soil passage beneath each cutoff tip, $\tau_j = [k_1 \max(t_1 - d_j, 0) + k_2(T - \max(t_1, d_j))]/(k_2 T)$, which removes the near-discontinuity that arises when a tip approaches the interface. The networks therefore learn bounded, scale-free functions instead of responses spanning five orders of magnitude. The learner is a deep ensemble [21] of six fully connected networks (four hidden layers of 256 SiLU units) trained on log-transformed targets by mini-batch AdamW followed by L-BFGS refinement; the ensemble mean is the prediction, and the member spread indicates epistemic uncertainty.

For the public database, the same principle was applied through the transformed-section analogy (horizontal lengths scaled by the square root of the anisotropy ratio) and the analytical exit-gradient distribution $i(s) = H/[\pi \sqrt{s(s + b')}]$ [3] evaluated at the first downstream grid cell. CatBoost [22], LightGBM, XGBoost [23] and Extra-Trees members were tuned by Optuna [24] on Models6 and averaged with equal weights (PG-Ens).

2.4. Benchmarks, evaluation and implementation

Seven conventional learners widely used in the literature (MLR, ANN, SVR, RF, XGBoost, LightGBM and CatBoost) were trained on the raw variables. All models used the same 80/20 train–test split; the test set was never used for model selection. Performance was measured by R^2 , RMSE, MAE, MAPE, the Kling–Gupta efficiency (KGE) and the a20-index, and SHAP values [25] were computed for interpretation. All computations were implemented in Python 3.11 using NumPy/SciPy, scikit-learn, XGBoost, LightGBM, CatBoost, PyTorch and SHAP.

3. Results and Discussion

3.1. Effect of soil layer permeability

Fig. 2 compares head fields for $k_1/k_2 = 0.01, 1$ and 100 . A less permeable upper layer behaves as a cap: the head loss concentrates within it, the lower layer becomes nearly equipotential and the exit gradient rises fourfold. A more permeable upper layer conveys most of the flow above the interface, so the downstream cutoff, whose tip lies just below the interface, dissipates a large share of the head; uplift therefore increases and the exit gradient falls.

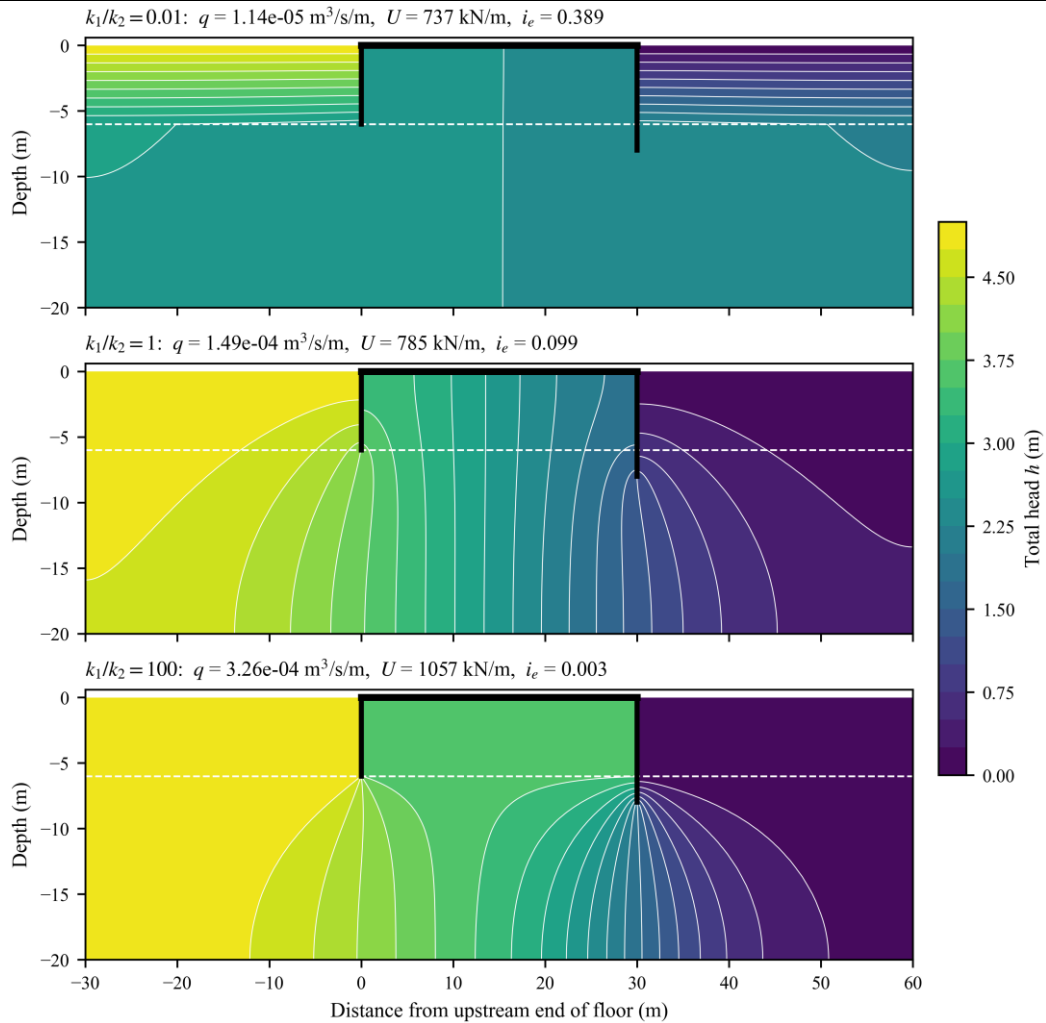


Fig. 2. Total-head fields for three permeability ratios ($H = 5 \text{ m}$, $b = 30 \text{ m}$, $T = 20 \text{ m}$, $d_1 = 6 \text{ m}$, $d_2 = 8 \text{ m}$, $t_1 = 6 \text{ m}$).

Fig. 3 generalises these trends. Discharge varies from 0.06 to 82 times the homogeneous value, and its sensitivity to k_1/k_2 grows with t_1/T because a thicker permeable layer provides a continuous flow path beneath both cutoffs. The exit gradient increases by up to 5.3 times when a thin, less permeable layer overlies a pervious stratum, which is the most critical condition for piping. Uplift changes by less than 7% except when $t_1 \approx d_2$, where it rises by 74%. These results reconcile the opposite trends reported earlier [4], [5]: the direction of the layer effect depends on which side of the interface the cutoff tips lie.

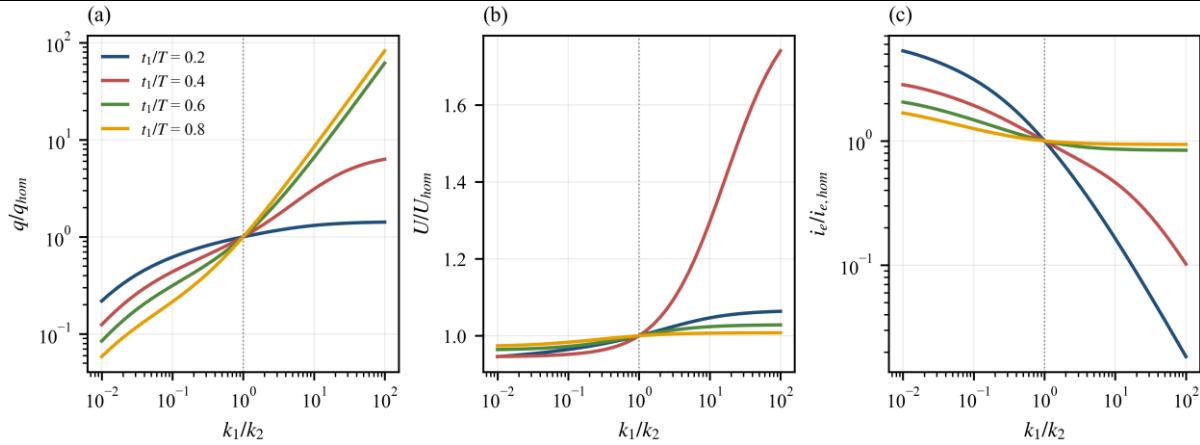


Fig. 3. Influence of k_1/k_2 and t_1/T on (a) discharge, (b) uplift and (c) exit gradient, normalised by the homogeneous case.

3.2. Predictive performance

Table 2 shows that conventional learners trained on raw variables cannot represent responses spanning several orders of magnitude: the best of them (CatBoost) attained $R^2 = 0.791$, 0.969 and 0.976, with MAPE of 739% for q . PG-DE reduced the RMSE by 98.1%, 96.4% and 93.8% relative to CatBoost and kept more than 99.8% of predictions within $\pm 20\%$ (a20-index). The parity plots (Fig. 4) show no systematic bias, including at the extreme permeability contrasts, and the mean spread among ensemble members ($\leq 0.35\%$) confirms a low epistemic uncertainty.

Table 2. Test-set performance on the two-layer database (3,277 unseen cases).

Model	$R^2 (q)$	$R^2 (U)$	$R^2 (i_e)$	MAPE q (%)	MAPE U (%)	MAPE i_e (%)
MLR	0.4055	0.8197	0.3126	18467.04	49.71	354.75
ANN	0.8762	0.9592	0.6764	15813.60	13.03	123.20
SVR	0.4131	0.8404	0.4955	101897.68	30.23	203.20
RF	0.7075	0.9304	0.9007	55.36	21.72	46.32
XGBoost	0.6371	0.9536	0.9318	580.22	15.92	57.90
LightGBM	0.7635	0.9531	0.9457	368.99	14.18	55.18
CatBoost	0.7912	0.9687	0.9765	739.11	14.63	42.82
PG-DE (proposed)	0.99992	0.99996	0.99991	0.32	0.20	0.43

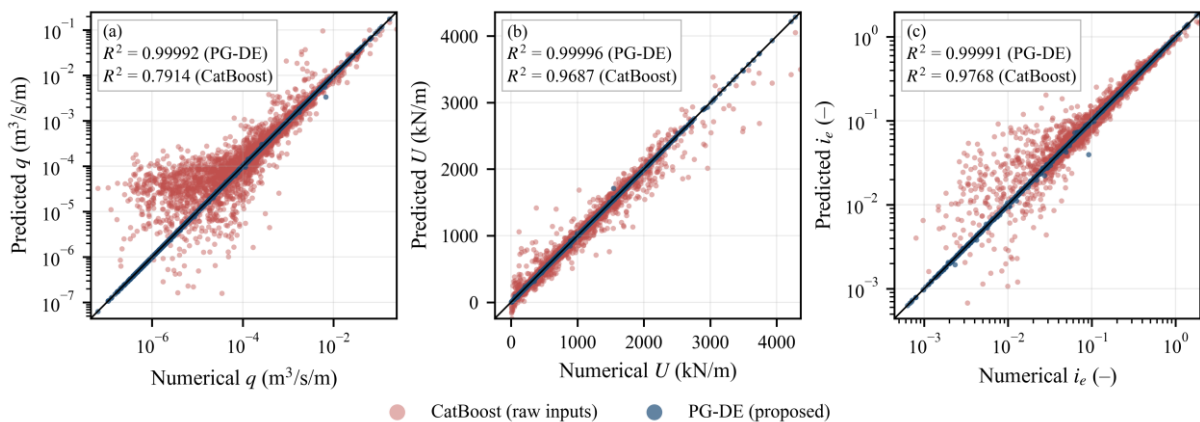


Fig. 4. Predicted versus numerical (a) q , (b) U and (c) i_e for the 3,277 test cases.

3.3. Benchmark on the public database

Under the original ten-fold, three-repeat cross-validation of Rath et al. [13], the reproduced XGBoost model gave $R^2 = 0.710$, matching their reported value of 0.71, whereas PG-Ens reached 0.902 (Table 3). On 40% hold-out tests the improvement persisted ($0.434 \rightarrow 0.872$ and $0.760 \rightarrow 0.919$). Because both models used identical data, the gain is attributable to the physics-guided features alone. The residual scatter (Fig. 5) reflects the coarse discretisation of the source model rather than a deficiency of the learner.

Table 3. Exit-gradient prediction on the public MODFLOW database.

Model	R^2 CV (Models5)	R^2 test (Models5)	R^2 test (Models5+7)	RMSE test (Models5+7)
XGBoost, Rath et al. (reported)	0.71	–	0.88 *	–
XGBoost, Rath et al. settings (reproduced)	0.694	0.434	0.760	4.872
XGBoost, default (raw inputs)	0.710	0.429	0.768	4.786
PG-CatBoost	0.901	0.871	0.897	3.184
PG-XGBoost	0.880	0.839	0.915	2.894
PG-Ens (proposed)	0.902	0.872	0.919	2.830

* Reported on the authors' 8,000-run set with 4,800 training cases; the proposed model used 2,399 training cases.

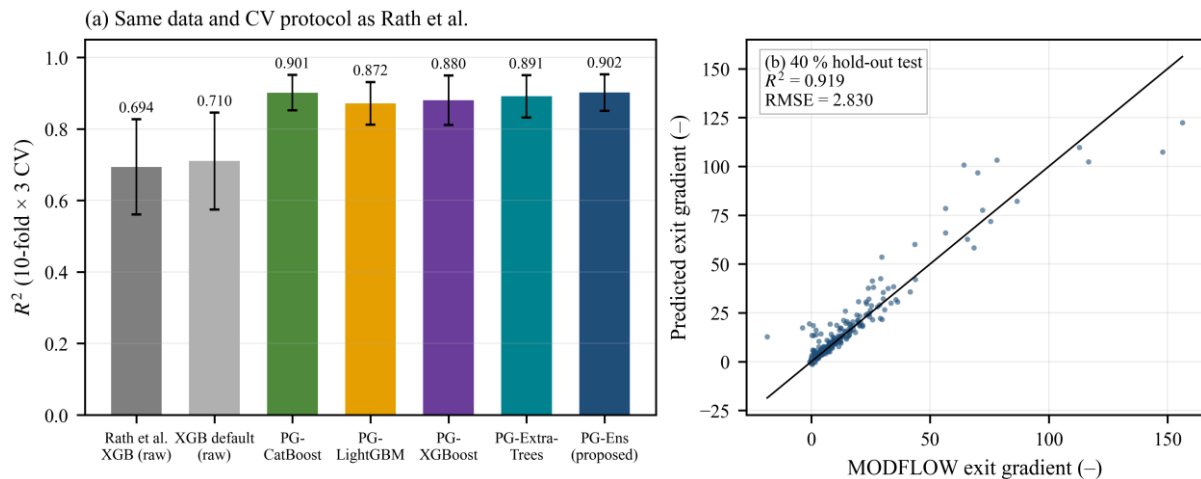


Fig. 5. Public-database benchmark: (a) cross-validated R^2 and (b) PG-Ens hold-out predictions (Models5+7).

3.4. Explainability

SHAP analysis (Fig. 6a) shows that the permeability-related groups (k_1/k_2 , the equivalent permeabilities and the passage transmissivities) account for 56% and 56% of the mean absolute attribution for q and i_e , with k_1/k_2 alone being the most influential input. Uplift is instead governed by the cutoff depths (43%) and by the position of their tips relative to the interface (24%), consistent with the threshold behaviour in Fig. 3b. Fig. 6b shows that decreasing k_1/k_2 raises the exit gradient monotonically, most strongly for thin upper layers, in agreement with Fig. 3c. These attributions are consistent with classical seepage theory [3], supporting the physical reliability of the surrogate.

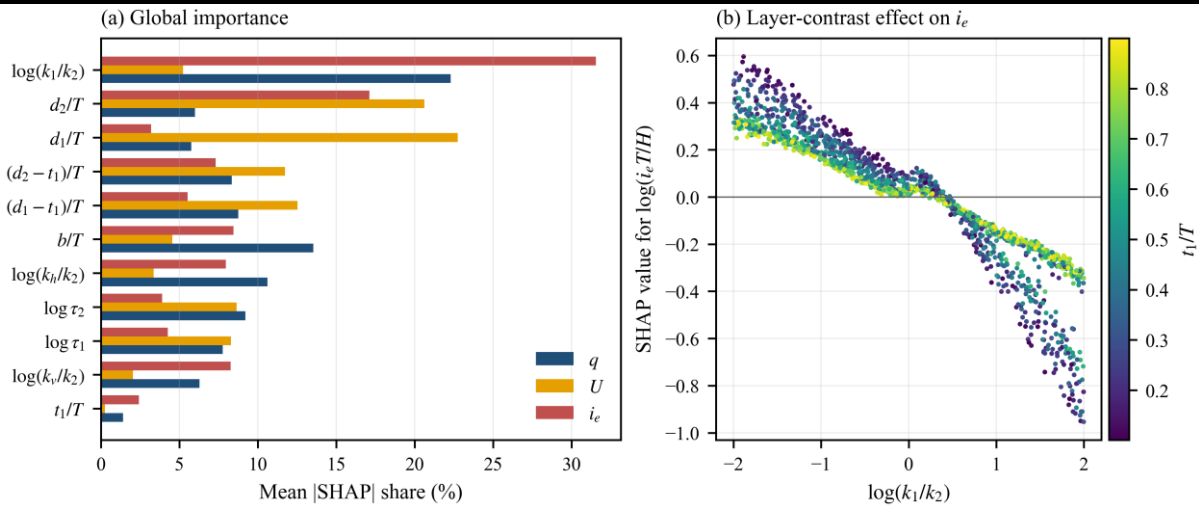


Fig. 6. SHAP interpretation: (a) global importance; (b) dependence of the exit gradient on k_1/k_2 .

3.5. Comparison with the state of the art

Table 4 compares the present results with recent surrogates for seepage beneath hydraulic structures. Direct comparison is indicative, because each study used its own data; nevertheless, PG-DE exceeds every coefficient of determination reported to three or four decimals and equals the only two-decimal value (1.00 for uplift [10]), while treating a more general two-layer problem with a database 2–300 times larger. Table 5 summarises the margins over the strongest prior result for each criterion.

TABLE 4. Comparison of the state-of-the-art work.

Ref.	Year	N	Foundation	Model	Target	Best test R^2
[14]	2021	1,000	Homogeneous, anisotropic	GEP	q	0.971
[15]	2021	>975	Homogeneous	GEP	i_e	0.957
[10]	2023	N/S	Homogeneous	FFNN	$U / q / i_e$	1.00 / 0.9994 / 0.9997
[12]	2025	630	Homogeneous	CatBoost	$U / i_e / q$	0.954 / 0.932 / 0.972
[13]	2025	8,000	Homogeneous, anisotropic	XGBoost	i_e	0.88
[16]	2025	90	Homogeneous	ANN-MLP	U (key points)	0.997–0.999
[11]	2026	55	Homogeneous	Ensemble NN	$U / q / i_e$	0.996 / 0.981 / 0.979
[17]	2026	77	Homogeneous	ANN-SSOA	$U_{\max}$	0.9982
This study	2026	16,384	Two layers ($k_1/k_2 = 0.01$ – 100)	PG-DE	$U / q / i_e$	0.99996 / 0.99992 / 0.99991

Table 5. Final summary: strengths of the proposed approach versus the best prior results.

Criterion	Best prior result	This study	Outcome
R^2 , seepage discharge q	0.9994 [10]	0.99992	Higher
R^2 , uplift U	1.00 (2 d.p.) [10]; 0.9982 [17]	0.99996	Equal at 2 d.p.; higher than all other studies
R^2 , exit gradient i_e	0.9997 [10]	0.99991	Higher
R^2 , i_e on Rath et al. data (CV)	0.71 [13]	0.902	+27%
R^2 , i_e on Rath et al. data (test)	0.88 [13]	0.919	+4.4%
MAPE (all targets)	0.72% (U only) [17]	$\leq 0.43\%$	All three targets
Database size	8,000 [13]	16,384	$2.0 \times$
Soil-layer permeability as input	Not considered	k_1, k_2, t_1	New capability
Interpretability	SHAP, single layer [13]	SHAP + similarity laws	Physically consistent

4. Conclusions

This study quantified the effect of soil layer permeability on seepage beneath hydraulic structures and developed an explainable physics-guided surrogate. The main conclusions are as follows. (1) Layer permeability is a first-order design variable: relative to homogeneous soil, discharge changed by factors of 0.06–82 and the exit gradient by up to 5.3, with the most critical condition being a thin, less permeable surface layer over a pervious stratum. (2) Uplift is insensitive to layering except when the downstream cutoff tip reaches the layer interface, where it increased by 74%. (3) Embedding dimensional similarity enabled PG-DE to predict q , U and i_e with $R^2 \geq 0.9999$ and $\text{MAPE} \leq 0.43\%$, outperforming seven conventional learners and exceeding all previously reported surrogates (equalling only a two-decimal uplift value of 1.00). (4) The same strategy improved exit-gradient prediction on an independent public database from $R^2 = 0.71$ to 0.902. Future work should extend the framework to three or more layers, inclined layers and transient conditions, and validate it against field piezometric data.

Data and Code Availability

The public MODFLOW database used for benchmarking is available at <https://doi.org/10.17632/mww9v4xg9x.1>. The two-layer finite-volume database generated in this study is available from the corresponding author on reasonable request. The models were implemented in Python 3.11 (scikit-learn, CatBoost, LightGBM, XGBoost, PyTorch and SHAP); the source code is available from the corresponding author on reasonable request.

References

- [1] E. W. Lane, "Security from under-seepage: masonry dams on earth foundations," *Trans. ASCE*, vol. 100, pp. 1235–1272, 1935.
- [2] A. N. Khosla, N. K. Bose, and E. M. Taylor, *Design of Weirs on Permeable Foundations*, Publ. no. 12. New Delhi, India: Central Board of Irrigation, 1936.
- [3] M. E. Harr, *Groundwater and Seepage*. New York, NY, USA: McGraw-Hill, 1962.
- [4] A. A. J. Jamel, "Effect of two sheet piles in double soil layers on seepage properties under hydraulic structure using SEEP/W program," *Al-Nahrain J. Eng. Sci.*, vol. 20, no. 1, pp. 194–205, 2017.
- [5] N. Mansour, N. M. AboulAtta, and D. A. El-Molla, "Control of seepage in multi-layered soil under heading-up structures using vertical and inclined sheet piles," *J. Al-Azhar Univ. Eng. Sector*, vol. 17, no. 62, pp. 96–117, 2022, doi: 10.21608/aej.2022.216591.
- [6] T. H. Nassralla and A. Rabea, "Seepage characteristics under hydraulic structure foundation (supported by sheet pile) in multi-layers soil," *Egypt. Int. J. Eng. Sci. Technol.*, vol. 18, no. 4, pp. 229–238, 2015, doi: 10.21608/eijest.2015.97121.
- [7] G. Kheiri, H. Javdanian, and G. Shams, "A numerical modeling study on the seepage under embankment dams," *Model. Earth Syst. Environ.*, vol. 6, pp. 1075–1087, 2020, doi: 10.1007/s40808-020-00742-9.
- [8] A. A. J. Jamel, "Effect of intermediate sheet piles in non-homogenous soil on seepage properties under hydraulic structure using SEEP/W program," *Tikrit J. Eng. Sci.*, vol. 23, no. 3, pp. 79–90, 2016, doi: 10.25130/tjes.23.3.09.
- [9] D. V. Griffiths and G. A. Fenton, "Probabilistic analysis of exit gradients due to steady seepage," *J. Geotech. Geoenviron. Eng.*, vol. 124, no. 9, pp. 789–797, 1998, doi: 10.1061/(ASCE)1090-0241(1998)124:9(789).
- [10] R. N. Alrowais, B. Alwushayh, M. T. Bashir, B. M. Nasef, A. Ghazy, and E. Elkamhaw, "Modeling and analysis of cutoff wall performance beneath water structures by feed-forward neural network (FFNN)," *Water*, vol. 15, no. 21, Art. no. 3870, 2023, doi: 10.3390/w15213870.
- [11] E. Elkamhaw, M. S. Sawah, M. Tawfik, and A. S. Ismail, "Ensemble neural network models for stability prediction and optimization of hydraulic structures considering uplift pressure and exit gradient," *Sci. Rep.*, vol. 16, Art. no. 1307, 2026, doi: 10.1038/s41598-025-25149-3.
- [12] M. K. Elshaarawy, M. Zelenáková, and A. M. Armanuos, "Hydraulic performance modeling of inclined double cutoff walls beneath hydraulic structures using optimized ensemble machine learning," *Sci. Rep.*, vol. 15, Art. no. 27592, 2025, doi: 10.1038/s41598-025-10990-3.

- [13] P. Rath, J. Zhu, and K. M. Befus, "Explainable machine learning surrogate modeling for exit gradient prediction in hydraulic structures using numerical simulations," *J. Hydroinform.*, vol. 27, no. 8, pp. 1309–1326, 2025, doi: 10.2166/hydro.2025.046.
- [14] A. Johari, A. Heydari, and A. Talebi, "Prediction of discharge flow rate beneath sheet piles using scaled boundary finite element modeling database," *Sci. Iran. A*, vol. 28, no. 2, pp. 645–655, 2021, doi: 10.24200/sci.2020.53281.3158.
- [15] D. H. Khashan and W. H. Hassan, "Studying the effect of head difference on exit gradients and uplift pressures beneath hydraulic structures by gene expression programming," *IOP Conf. Ser.: Mater. Sci. Eng.*, vol. 1094, Art. no. 012099, 2021, doi: 10.1088/1757-899X/1094/1/012099.
- [16] B. Nourani, S. Kamalinezhad, F. Salmasi, S. Roumianfar, and M. Ansari, "Estimating the uplift pressure under hydraulic structures with dual end cutoff walls with finite element, regression and intelligent approaches," *Hydraul. Water Sci.*, vol. 35, no. 4, pp. 45–58, 2025, doi: 10.22034/hws.2026.70179.1038.
- [17] A. R. Mazaheri, Y. S. Azimi, M. Komasi, and B. Beiranvand, "Single-objective geometry optimization of cutoff walls in concrete gravity dam foundations using a surrogate-assisted FEM–ANN–SSOA framework," *Sci. Rep.*, 2026, doi: 10.1038/s41598-026-66384-6.
- [18] M. M. Sayed Ahmed, M. Z. Khursheed, B. Alshameri, A. K. Hamed, and M. K. Elshaarawy, "Estimating seepage in heterogeneous earthfill dams on permeable foundations using explainable machine learning," *Sci. Rep.*, vol. 16, Art. no. 12060, 2026, doi: 10.1038/s41598-026-45048-5.
- [19] P. Rath, "Exit gradient generation using Modflow and implementation in XGBoost," Mendeley Data, V1, 2025, doi: 10.17632/mww9v4xg9x.1.
- [20] E. Buckingham, "On physically similar systems; illustrations of the use of dimensional equations," *Phys. Rev.*, vol. 4, no. 4, pp. 345–376, 1914, doi: 10.1103/PhysRev.4.345.
- [21] B. Lakshminarayanan, A. Pritzel, and C. Blundell, "Simple and scalable predictive uncertainty estimation using deep ensembles," in *Proc. Adv. Neural Inf. Process. Syst. (NeurIPS)*, vol. 30, 2017, pp. 6402–6413.
- [22] L. Prokhorenkova, G. Gusev, A. Vorobev, A. V. Dorogush, and A. Gulin, "CatBoost: unbiased boosting with categorical features," in *Proc. Adv. Neural Inf. Process. Syst. (NeurIPS)*, vol. 31, 2018, pp. 6638–6648.
- [23] T. Chen and C. Guestrin, "XGBoost: a scalable tree boosting system," in *Proc. 22nd ACM SIGKDD Int. Conf. Knowl. Discov. Data Min.*, 2016, pp. 785–794, doi: 10.1145/2939672.2939785.
- [24] T. Akiba, S. Sano, T. Yanase, T. Ohta, and M. Koyama, "Optuna: a next-generation hyperparameter optimization framework," in *Proc. 25th ACM SIGKDD Int. Conf. Knowl. Discov. Data Min.*, 2019, pp. 2623–2631, doi: 10.1145/3292500.3330701.
- [25] S. M. Lundberg and S.-I. Lee, "A unified approach to interpreting model predictions," in *Proc. Adv. Neural Inf. Process. Syst. (NeurIPS)*, vol. 30, 2017, pp. 4765–4774.