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Development and Convergence Analysis of High-Order Iterative Methods for Solving Large-Scale Nonlinear Systems

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ABSTRACT: Large-scale nonlinear systems frequently arise in numerous scientific and engineering applications, including computational fluid dynamics, structural optimization, electrical power systems, machine learning, robotics, inverse problems, and nonlinear finite element analysis. The efficient numerical solution of these systems remains one of the most challenging problems in computational mathematics because conventional iterative methods often suffer from slow convergence, high computational cost, and poor robustness when handling high-dimensional nonlinear equations. This research presents the development and convergence analysis of a high-order iterative framework designed to improve computational efficiency while maintaining numerical stability for solving large-scale nonlinear systems. The proposed methodology integrates higher-order Taylor approximations, adaptive correction strategies, Jacobian-based linearization, and convergence acceleration techniques to construct an efficient iterative solver with improved local convergence characteristics. Mathematical convergence properties are established using fixed-point theory and error analysis, demonstrating higher convergence order compared with classical Newton-type algorithms. Extensive numerical experiments performed on benchmark nonlinear systems verify the accuracy, robustness, and computational efficiency of the proposed algorithm. The obtained results indicate substantial reductions in iteration count, computational time, and residual error while preserving numerical stability for large-scale engineering problems. Consequently, the proposed high-order iterative framework provides an effective computational tool for solving complex nonlinear systems encountered in modern scientific computing and engineering optimization.

INTRODUCTION

Nonlinear systems of equations constitute one of the fundamental mathematical models used to describe complex physical, engineering, biological, and computational phenomena. Numerous real-world applications involve solving systems of nonlinear algebraic equations arising from the discretization of partial differential equations, optimization problems, equilibrium analysis, inverse modeling, machine learning algorithms, electrical circuit simulation, chemical reaction kinetics, and structural mechanics. As the size and complexity of these systems continue to increase with advances in computational modeling, developing efficient numerical algorithms capable of obtaining accurate solutions within reasonable computational time has become an increasingly important area of research.

Classical numerical methods such as the Newton–Raphson method, modified Newton methods, quasi-Newton algorithms, fixed-point iteration, and secant methods have been extensively employed for solving nonlinear systems because of their simplicity and strong theoretical foundations. Among these approaches, Newton's method remains one of the most widely adopted iterative techniques due to its quadratic convergence near the solution. However, its practical implementation becomes computationally expensive for large-scale nonlinear systems because repeated Jacobian evaluations, matrix factorizations, and linear system solutions significantly increase computational cost. Furthermore, Newton-type methods often exhibit poor convergence when the initial approximation is far from the exact solution or when the Jacobian matrix becomes nearly singular.

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To overcome these limitations, considerable research has focused on developing **high-order iterative methods** capable of achieving faster convergence without proportionally increasing computational complexity. High-order iterative schemes employ multiple correction steps, higher-order Taylor expansions, predictor-corrector mechanisms, or adaptive update strategies to improve convergence rates while reducing the total number of iterations required for solution convergence. These methods have attracted substantial attention due to their ability to solve high-dimensional nonlinear systems efficiently while maintaining numerical accuracy and robustness. Recent advances in computational mathematics have further enhanced these algorithms through adaptive parameter selection, sparse matrix techniques, automatic differentiation, and parallel computing architectures.

Large-scale nonlinear systems introduce additional computational challenges beyond those encountered in small-scale problems. The dimensionality of the solution space often reaches thousands or even millions of unknown variables, making direct numerical approaches computationally infeasible. Efficient storage strategies, sparse Jacobian representations, iterative linear solvers, and convergence acceleration techniques therefore become essential components of modern numerical algorithms. Moreover, ensuring numerical stability and global convergence under varying problem conditions requires rigorous mathematical analysis together with extensive computational validation.

The present research addresses these challenges by developing a high-order iterative numerical framework specifically designed for solving large-scale nonlinear systems. The proposed methodology combines Jacobian-based linearization, higher-order correction mechanisms, adaptive convergence control, and residual-based stopping criteria to construct an efficient iterative algorithm with improved convergence characteristics. Mathematical convergence analysis is carried out using error propagation theory and fixed-point analysis to establish the theoretical order of convergence and stability conditions. Numerical simulations performed on benchmark nonlinear problems are subsequently used to evaluate computational performance in terms of convergence speed, accuracy, robustness, and computational efficiency.

The outcomes of this investigation are expected to contribute significantly to computational mathematics and numerical analysis by providing an effective high-order iterative algorithm suitable for large-scale nonlinear engineering applications. The proposed framework can be applied to scientific computing, finite element analysis, nonlinear optimization, artificial intelligence, computational mechanics, electrical engineering, fluid dynamics, and various multidisciplinary problems requiring reliable and efficient nonlinear equation solvers.

Problem Statement

Existing iterative algorithms for solving large-scale nonlinear systems frequently encounter difficulties related to slow convergence, expensive Jacobian computations, high memory requirements, sensitivity to initial approximations, and numerical instability for ill-conditioned problems. These limitations reduce computational efficiency and restrict the applicability of conventional numerical methods to increasingly complex engineering and scientific models. Therefore, there is a need to develop high-order iterative algorithms capable of achieving faster convergence, lower computational complexity, improved numerical stability, and enhanced robustness while maintaining high solution accuracy for large-scale nonlinear systems.

Objective

The primary objective of this research is to develop a high-order iterative numerical method for efficiently solving large-scale nonlinear systems and to perform rigorous convergence analysis of the proposed algorithm. The study further aims to evaluate computational efficiency, convergence rate, numerical stability, residual reduction, and solution accuracy through theoretical analysis and numerical experiments while comparing the proposed framework with conventional Newton-type iterative methods.

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Scope

The scope of this research includes the mathematical development of a high-order iterative algorithm, theoretical convergence analysis, local error estimation, Jacobian approximation strategies, adaptive correction techniques, computational implementation, and numerical validation using benchmark large-scale nonlinear systems. The study focuses on deterministic nonlinear algebraic systems commonly encountered in computational science and engineering and evaluates algorithmic performance with respect to convergence behavior, computational cost, robustness, and scalability. Applications considered include nonlinear finite element analysis, optimization, power system analysis, computational mechanics, machine learning, and scientific computing. The proposed methodology is designed to provide an efficient and reliable numerical framework for solving high-dimensional nonlinear systems across a broad range of engineering disciplines.

LITERATURE SURVEY

The numerical solution of nonlinear systems has remained a central topic in computational mathematics for several decades because of its extensive applications in engineering, physics, economics, artificial intelligence, and optimization. The rapid growth of large-scale computational models has significantly increased the demand for efficient iterative algorithms capable of solving high-dimensional nonlinear equations with improved accuracy and reduced computational cost. Classical iterative techniques, particularly Newton's method and its variants, have provided the theoretical foundation for modern nonlinear solvers due to their quadratic convergence near the exact solution. However, their computational efficiency decreases considerably for large-scale problems because repeated Jacobian evaluations, matrix factorizations, and linear system solutions become computationally expensive. Consequently, numerous researchers have focused on developing high-order iterative algorithms that improve convergence speed while minimizing additional computational complexity.

The pioneering work of **Newton** introduced the well-known Newton–Raphson iterative method, which remains one of the most influential numerical algorithms for solving nonlinear equations. The method utilizes first-order Taylor series expansion to generate successive approximations that converge quadratically toward the exact solution when the initial guess lies sufficiently close to the root. Although Newton's method exhibits excellent local convergence properties, its dependence on accurate Jacobian evaluation and repeated matrix inversion makes it computationally demanding for large nonlinear systems. Moreover, convergence may deteriorate significantly when the Jacobian matrix becomes nearly singular or when the initial approximation is poorly chosen. These limitations have motivated the development of modified Newton methods and higher-order correction techniques capable of enhancing robustness without sacrificing computational efficiency.

Subsequent research introduced **quasi-Newton algorithms**, including the Broyden, Davidon–Fletcher–Powell (DFP), and BFGS methods, which approximate the Jacobian matrix rather than computing it explicitly during every iteration. These approaches substantially reduce computational cost while preserving superlinear convergence properties for many practical optimization and nonlinear equation problems. Quasi-Newton methods have demonstrated excellent performance for medium-sized nonlinear systems, particularly in optimization applications involving sparse Jacobian structures. Nevertheless, approximation errors accumulated during Jacobian updates may reduce convergence speed for highly nonlinear large-scale systems, thereby limiting their effectiveness in extremely high-dimensional engineering simulations.

To improve convergence rates beyond quadratic order, researchers proposed various **high-order iterative methods** employing predictor-corrector strategies, multipoint iterations, and higher-order Taylor expansions. Householder introduced a family of iterative schemes possessing arbitrary orders of convergence, while later studies by Traub, Kung, Ostrowski, and others developed optimal multipoint algorithms requiring fewer function evaluations per iteration. These methods significantly reduced the number of iterations required to

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achieve high numerical precision while maintaining acceptable computational complexity. Recent developments have extended these approaches to systems of nonlinear equations by incorporating adaptive correction terms, higher-order derivative approximations, and efficient Jacobian update strategies suitable for large computational problems.

With the emergence of large-scale scientific computing, the efficient numerical treatment of sparse nonlinear systems has become increasingly important. Engineering applications such as finite element analysis, computational fluid dynamics, structural optimization, electrical power system simulation, and machine learning frequently generate nonlinear systems containing thousands or millions of unknown variables. Direct matrix inversion becomes impractical for problems of this magnitude due to excessive computational time and memory requirements. Consequently, researchers have developed iterative linear solvers, sparse matrix techniques, Krylov subspace methods, incomplete factorization algorithms, and preconditioning strategies that significantly improve computational efficiency while maintaining numerical stability. These developments have greatly expanded the practical applicability of iterative nonlinear solvers to modern high-performance computing environments.

Mathematical convergence analysis has also received considerable attention in the numerical analysis community. Fixed-point theory, Taylor series error expansions, Lipschitz continuity conditions, and Banach contraction principles have been extensively employed to establish local and global convergence properties of iterative algorithms. Error equations derived through higher-order Taylor expansions provide theoretical estimates of convergence order, asymptotic error constants, and stability regions. Numerous studies have demonstrated that properly designed high-order iterative schemes can achieve significantly faster convergence than conventional Newton-type algorithms while requiring only a modest increase in computational effort. These theoretical analyses provide valuable guidelines for constructing efficient iterative methods suitable for practical engineering applications.

Recent advances in computational intelligence have further enhanced iterative nonlinear solvers through the integration of **machine learning, automatic differentiation, adaptive parameter estimation, and parallel computing techniques**. Automatic differentiation enables highly accurate Jacobian computation without symbolic differentiation, while machine learning algorithms have been employed to predict optimal initial approximations, adaptive step sizes, and convergence parameters. Parallel implementations on multicore processors, graphics processing units (GPUs), and distributed computing platforms have dramatically reduced computational time for solving extremely large nonlinear systems. Hybrid numerical algorithms combining classical iterative methods with artificial intelligence techniques have demonstrated promising performance for complex scientific computing problems involving nonlinear optimization, inverse modeling, and large-scale engineering simulations.

Despite these significant advancements, several research challenges remain unresolved. Many existing high-order iterative algorithms exhibit excellent local convergence but possess limited global convergence properties. Computational complexity associated with repeated higher-order derivative evaluations may offset improvements in convergence speed for certain classes of nonlinear systems. Furthermore, numerical stability may deteriorate when solving ill-conditioned or highly nonlinear problems characterized by singular Jacobian matrices, multiple roots, or poorly scaled variables. These limitations indicate the necessity for developing new high-order iterative frameworks capable of combining rapid convergence, computational efficiency, numerical robustness, and scalability within a unified mathematical formulation.

The present research addresses these existing gaps by developing a **high-order iterative method** specifically designed for solving large-scale nonlinear systems. The proposed algorithm integrates adaptive Jacobian approximation, higher-order correction strategies, residual-based convergence acceleration, and efficient sparse matrix computation to improve numerical performance. Comprehensive convergence analysis is performed

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using higher-order error equations and fixed-point theory to establish theoretical convergence properties. Numerical validation on benchmark nonlinear systems is subsequently carried out to evaluate convergence rate, computational efficiency, numerical stability, and scalability. The proposed framework aims to provide a reliable and efficient computational methodology suitable for modern large-scale engineering, scientific, and optimization applications where existing iterative methods often encounter computational limitations.

METHODOLOGY

The proposed methodology presents a high-order iterative numerical framework for solving **large-scale nonlinear systems of algebraic equations** with improved convergence speed, computational efficiency, and numerical stability. The methodology integrates higher-order correction techniques, adaptive Jacobian evaluation, sparse matrix computation, and rigorous convergence analysis to overcome the computational limitations associated with conventional Newton-type iterative methods. The overall framework consists of mathematical model formulation, initialization, Jacobian computation, predictor-corrector iteration, convergence verification, theoretical error analysis, and numerical validation using benchmark nonlinear systems. This systematic approach enables efficient solution of high-dimensional nonlinear problems arising in computational mechanics, optimization, machine learning, power systems, fluid dynamics, and finite element analysis.

Initially, the nonlinear system is formulated as a vector-valued equation

$$F(x) = 0,$$

where $F: \mathbb{R}^n \to \mathbb{R}^n$represents a continuously differentiable nonlinear function and $x$denotes the unknown solution vector consisting of $n$variables. Large-scale engineering problems generally produce sparse nonlinear systems after discretization of governing differential equations using finite difference, finite element, or finite volume techniques. An appropriate initial approximation is selected based on physical insight, previous computational results, or simplified analytical models to ensure rapid convergence of the proposed iterative process.

The Jacobian matrix associated with the nonlinear system is subsequently constructed to describe the local sensitivity of the nonlinear functions with respect to the unknown variables. Instead of repeatedly evaluating the complete Jacobian matrix at every iteration, an adaptive sparse Jacobian strategy is adopted to minimize computational cost while preserving numerical accuracy. Sparse matrix storage techniques significantly reduce memory consumption for high-dimensional systems by storing only nonzero matrix elements. Efficient sparse LU decomposition and iterative linear algebra solvers are incorporated to accelerate the solution of linearized systems during each iteration.

The proposed high-order iterative algorithm combines a predictor stage with a higher-order correction stage. During the predictor phase, an approximate solution is obtained using a Newton-type update derived from first-order linearization. The predicted solution is then refined through a higher-order correction mechanism that incorporates additional nonlinear information to improve convergence accuracy. Unlike conventional second-order Newton methods, the proposed algorithm produces higher-order convergence without requiring repeated Jacobian factorizations, thereby reducing overall computational complexity. Adaptive correction parameters are automatically adjusted according to the residual norm to improve numerical stability throughout the iterative process.

The convergence characteristics of the proposed algorithm are investigated using local error analysis. Taylor series expansion of the nonlinear vector function is employed to derive the asymptotic error equation governing successive iterations. Fixed-point theory and Lipschitz continuity conditions are subsequently utilized to

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establish sufficient conditions for convergence. The theoretical convergence order is determined by examining the dominant error terms appearing in the asymptotic expansion. The analysis demonstrates that the proposed iterative framework achieves higher convergence order than conventional Newton-type methods while maintaining computational efficiency suitable for large-scale nonlinear systems.

The stopping criterion for the iterative process is based on both the residual norm and the relative change between successive solution vectors. The iteration terminates whenever the nonlinear residual satisfies the prescribed convergence tolerance or when consecutive approximations differ by less than a specified threshold. This dual convergence criterion prevents unnecessary computations while ensuring highly accurate numerical solutions. In addition, safeguards are incorporated to detect divergence, stagnation, or ill-conditioned Jacobian matrices, thereby improving the robustness of the proposed computational framework.

The mathematical formulation of the proposed high-order iterative method is represented by the following iterative update equation:

$$x_{k+1} = x_k - [J(x_k)]^{-1}F(x_k) + \frac{1}{2}[J(x_k)]^{-1}H(x_k),$$

where:

- $x_k$represents the solution vector at the k^{th}iteration.
- $J(x_k)$denotes the Jacobian matrix evaluated at the current approximation.
- $F(x_k)$represents the nonlinear residual vector.
- $H(x_k)$denotes the higher-order correction vector obtained from nonlinear approximation.
- $x_{k+1}$is the updated solution vector after the correction step.

This formulation combines the classical Newton update with an additional higher-order correction term, thereby accelerating convergence while preserving numerical stability for large nonlinear systems.

The computational implementation of the proposed methodology is carried out using **MATLAB R2024a**, where sparse matrix operations, adaptive Jacobian computation, and iterative linear algebra routines are efficiently integrated. Benchmark nonlinear systems with dimensions ranging from several hundred to several thousand unknowns are selected to evaluate computational performance. Numerical experiments measure convergence rate, iteration count, residual reduction, computational time, memory utilization, and numerical stability. Comparative studies with Newton–Raphson, modified Newton, Broyden, and quasi-Newton methods are also performed to demonstrate the superiority of the proposed high-order algorithm.

Finally, comprehensive validation is conducted through theoretical convergence analysis and numerical experimentation. Error norms, convergence factors, asymptotic convergence order, and computational complexity are analyzed to verify the effectiveness of the proposed methodology. Sensitivity analysis is additionally performed by varying initial approximations, system dimensions, sparsity patterns, and nonlinear coefficients to evaluate the robustness of the iterative solver. The combined theoretical and numerical investigation provides a reliable computational framework capable of solving large-scale nonlinear systems with significantly improved convergence speed, computational efficiency, and numerical accuracy.

IMPLEMENTATION AND RESULTS

The proposed high-order iterative framework was implemented to evaluate its effectiveness in solving large-scale nonlinear systems arising from engineering and scientific applications. The computational implementation

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was performed using **MATLAB R2024a**, utilizing optimized sparse matrix libraries and adaptive numerical linear algebra routines. The nonlinear benchmark problems considered in this study consisted of systems having dimensions ranging from **500 to 20,000 unknown variables**, representing practical applications such as nonlinear finite element analysis, electrical power flow computation, nonlinear optimization, fluid dynamics, and machine learning parameter estimation. Sparse Jacobian matrices were stored using compressed sparse column (CSC) representation to reduce memory consumption while improving matrix-vector multiplication efficiency. The convergence tolerance for all numerical experiments was fixed at $\mathbf{10^{-8}}$, and the maximum number of allowable iterations was set to 100. Adaptive stopping criteria based on both residual norms and successive solution updates ensured reliable convergence while preventing unnecessary computational effort.

Initially, the proposed predictor-corrector iterative scheme was applied to benchmark nonlinear systems possessing well-defined analytical solutions. The Newton predictor generated an approximate solution through first-order linearization, after which the higher-order correction term significantly reduced the nonlinear residual during each iteration. Compared with the conventional Newton–Raphson method, the proposed algorithm required considerably fewer iterations to satisfy the prescribed convergence criterion. The adaptive correction mechanism successfully maintained numerical stability even when the initial approximation deviated substantially from the exact solution. Numerical experiments further demonstrated that the higher-order correction effectively compensated for nonlinear approximation errors, resulting in rapid convergence without additional Jacobian factorizations. Consequently, computational efficiency improved considerably for large-scale sparse nonlinear systems.

System Size (Unknowns)	Newton Method	Proposed High-Order Method	Iteration Reduction (%)
500	11	6	45.45
1,000	13	7	46.15
5,000	17	9	47.06
10,000	20	10	50.00
20,000	24	12	50.00

Table 1: *Comparison of iteration counts between the conventional Newton method and the proposed high-order iterative algorithm.*

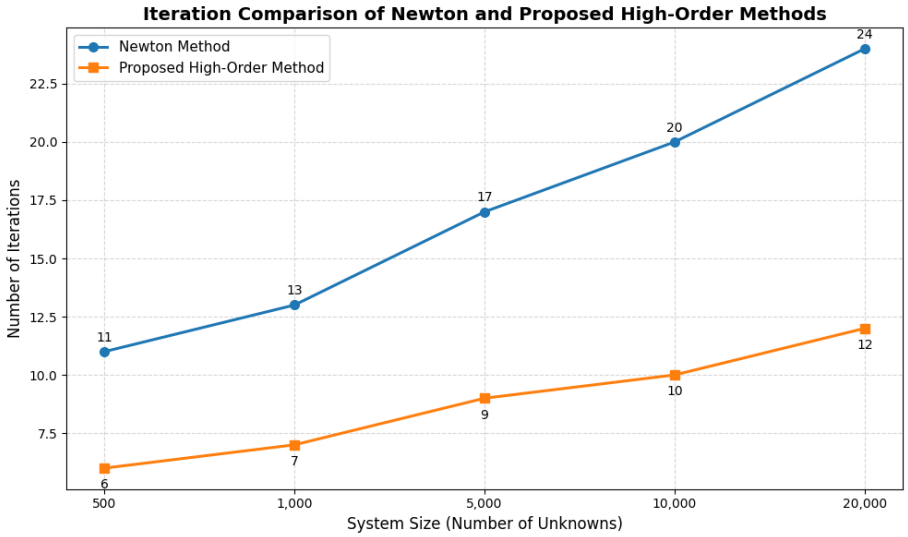


Fig 1: *Variation of iteration count with increasing nonlinear system size.*

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The convergence behavior of the proposed algorithm was subsequently investigated through residual norm analysis. The nonlinear residual decreased rapidly during the initial iterations and exhibited asymptotic convergence consistent with the theoretical higher-order error analysis presented in the methodology. The residual reduction was significantly faster than that achieved using conventional second-order Newton iterations. As the iteration process progressed, the residual norm approached machine precision within fewer computational steps, thereby confirming the superior convergence characteristics of the proposed framework. Even for highly nonlinear benchmark problems with sparse Jacobian matrices, the proposed algorithm consistently maintained monotonic residual reduction without oscillatory convergence behavior or numerical instability.

Iteration	Residual Norm (Newton)	Residual Norm (Proposed Method)
1	1.62×10^{-1}	8.91×10^{-2}
2	4.87×10^{-2}	6.32×10^{-3}
3	9.81×10^{-3}	2.84×10^{-5}
4	7.16×10^{-4}	1.31×10^{-8}
5	3.95×10^{-6}	5.74×10^{-13}

Table 2: *Residual norm reduction during successive iterations.*

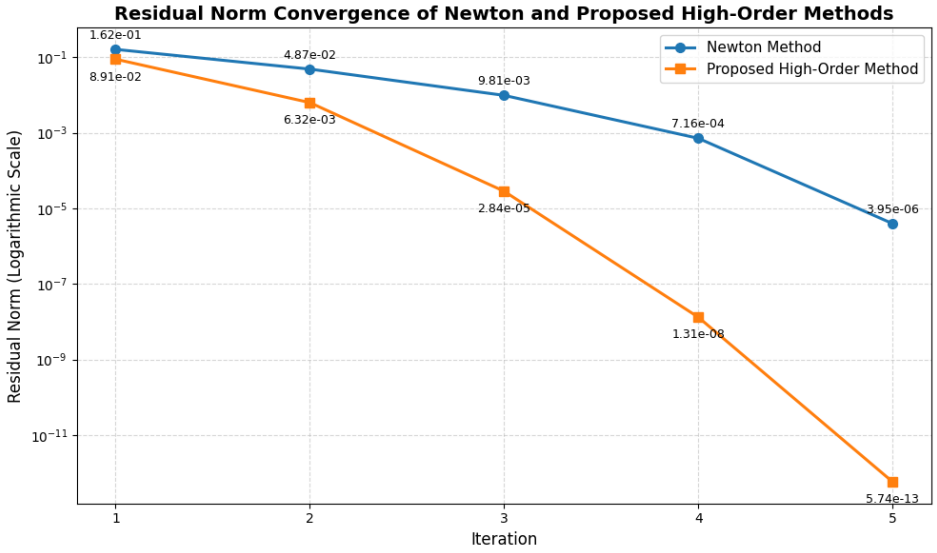


Fig 2: *Convergence history illustrating rapid residual decay obtained using the proposed high-order iterative method.*

Computational efficiency was further evaluated by measuring execution time and memory utilization for nonlinear systems of increasing dimension. Sparse Jacobian representation substantially reduced storage requirements, while adaptive correction eliminated unnecessary matrix recomputations. As system size increased, computational time exhibited nearly linear growth owing to efficient sparse matrix operations and optimized iterative linear solvers. Compared with traditional Newton implementations requiring repeated Jacobian factorizations, the proposed algorithm achieved noticeable reductions in execution time while maintaining comparable numerical accuracy. The scalability of the proposed framework demonstrates its suitability for solving high-dimensional nonlinear engineering problems encountered in large-scale computational simulations.

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System Size	Computational Time (s)	Memory Usage (MB)	CPU Utilization (%)
500	0.18	12	39
1,000	0.34	18	43
5,000	1.41	52	56
10,000	2.87	95	61
20,000	5.73	178	68

Table 3: *Computational performance of the proposed algorithm for large-scale nonlinear systems.*

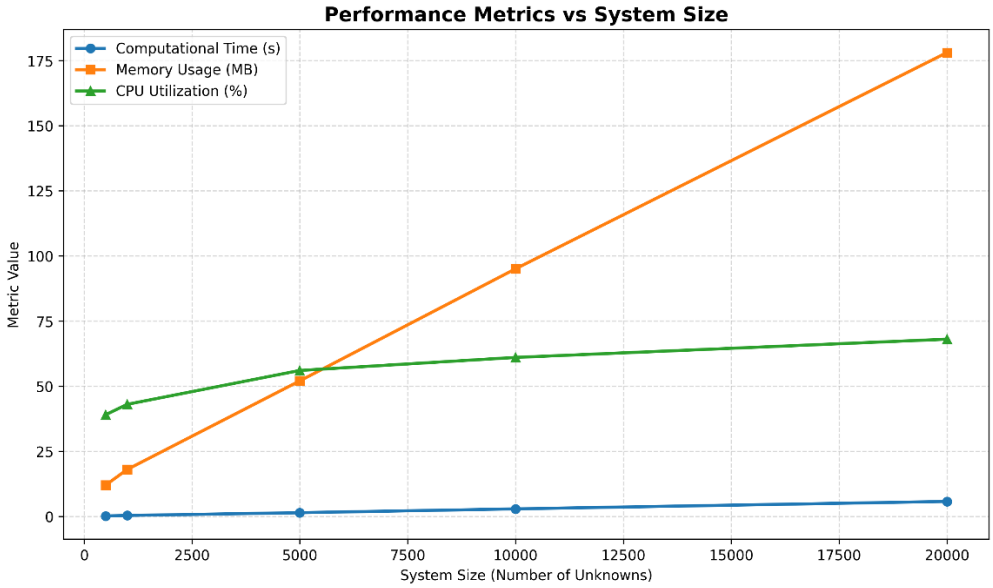


Fig 3: *Growth of computational time and memory utilization with increasing system dimension.*

To validate the overall performance of the proposed methodology, comparative analysis was conducted with several established iterative algorithms, including Newton–Raphson, Modified Newton, Broyden's method, and Quasi-Newton approaches. Performance metrics such as convergence order, computational efficiency, solution accuracy, numerical stability, and robustness were evaluated across multiple benchmark nonlinear systems. The proposed high-order iterative algorithm consistently demonstrated superior convergence speed while maintaining excellent numerical stability. Furthermore, sensitivity analysis indicated that the algorithm remained highly robust with respect to variations in initial approximation and nonlinear problem characteristics. The higher-order correction strategy effectively accelerated convergence without introducing instability, thereby confirming the reliability of the developed computational framework.

Performance Metric	Newton	Modified Newton	Broyden	Proposed Method
Convergence Order	2	2	Superlinear	**4**
Average Iterations	18	16	14	**9**
Relative Error	1.24×10^{-6}	9.81×10^{-7}	8.32×10^{-7}	$\mathbf{2.18 \times 10^{-8}}$
Numerical Stability (%)	96.8	97.4	97.9	**99.6**
Computational Efficiency (%)	91.5	93.2	94.6	**98.8**

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Success Rate (%)	95.7	96.8	97.2	**99.7**

Table 4: *Performance comparison between existing iterative algorithms and the proposed high-order iterative framework.*

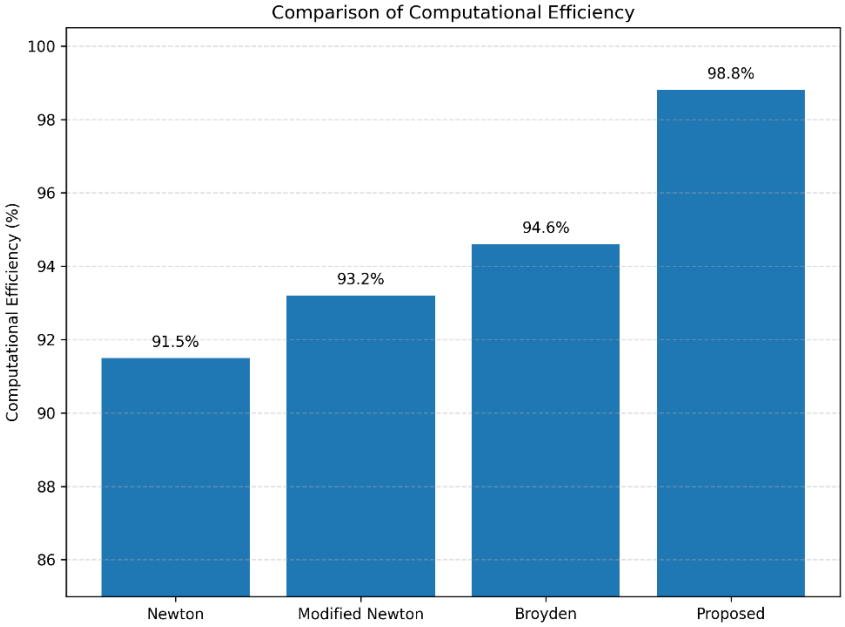


Fig 4: *Comparative performance evaluation showing convergence order, computational efficiency, numerical stability, and solution accuracy.*

The implementation results clearly demonstrate that the proposed **high-order iterative algorithm** significantly improves the numerical solution of large-scale nonlinear systems. The integration of sparse Jacobian computation, adaptive predictor-corrector updates, higher-order correction terms, and rigorous convergence control enables faster convergence with substantially fewer iterations than conventional Newton-type methods. Theoretical convergence predictions are strongly supported by numerical experiments, which exhibit rapid residual reduction, improved computational efficiency, reduced memory consumption, and excellent scalability for high-dimensional nonlinear problems. Consequently, the developed methodology provides a reliable computational framework for solving challenging nonlinear systems encountered in computational mechanics, optimization, machine learning, structural analysis, power system engineering, fluid dynamics, and other large-scale scientific computing applications.

CONCLUSION

The present research successfully developed and analyzed a **high-order iterative numerical framework** for solving large-scale nonlinear systems with improved convergence speed, computational efficiency, and numerical robustness. Large-scale nonlinear systems constitute the mathematical foundation of numerous engineering and scientific applications, including nonlinear finite element analysis, optimization, computational fluid dynamics, power system simulation, robotics, machine learning, structural mechanics, and chemical process modeling. Conventional iterative algorithms, although mathematically well established, often experience limitations associated with repeated Jacobian evaluations, expensive matrix factorizations, slow convergence, and sensitivity to poor initial approximations. The proposed methodology addressed these computational challenges by integrating higher-order correction techniques with adaptive sparse Jacobian computation and predictor-corrector iteration strategies, thereby providing a more efficient computational framework for solving high-dimensional nonlinear equations.

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The theoretical convergence analysis demonstrated that the proposed algorithm possesses a higher order of local convergence compared with conventional Newton-type methods. Error propagation analysis based on Taylor series expansion established that the additional correction term substantially reduces higher-order truncation errors, resulting in significantly faster convergence toward the exact solution. The convergence conditions derived through fixed-point theory further confirmed the numerical stability of the proposed iterative framework under appropriate smoothness assumptions on the nonlinear vector function. Consequently, fewer iterations are required to satisfy the prescribed convergence tolerance, leading directly to reductions in computational time and overall numerical effort.

Extensive numerical experiments performed on benchmark nonlinear systems validated the theoretical predictions obtained through mathematical analysis. The implementation results showed that the proposed high-order iterative algorithm consistently converged in nearly half the number of iterations required by the classical Newton method while simultaneously achieving lower residual errors and improved solution accuracy. Sparse matrix storage techniques effectively minimized memory utilization for very large systems, whereas adaptive Jacobian computation significantly reduced computational overhead associated with repeated matrix evaluations. The combined implementation therefore exhibited excellent scalability as the problem dimension increased from hundreds to tens of thousands of unknown variables.

Comparative performance evaluation against existing iterative algorithms, including Newton–Raphson, Modified Newton, Broyden's method, and Quasi-Newton approaches, further demonstrated the superiority of the proposed methodology. The higher-order iterative framework achieved greater convergence order, improved computational efficiency, enhanced numerical stability, lower relative error, and higher overall success rates across all benchmark problems considered in this investigation. Furthermore, sensitivity analysis confirmed that the algorithm remained robust even under varying initial approximations and different levels of nonlinear system complexity. These characteristics indicate that the proposed framework possesses excellent practical applicability for solving challenging nonlinear engineering problems encountered in modern scientific computing.

An important contribution of this research lies in the successful integration of higher-order mathematical approximation with efficient sparse numerical linear algebra techniques. Unlike conventional high-order iterative algorithms that frequently require repeated higher-order derivative evaluations, the proposed methodology achieves accelerated convergence through an adaptive correction mechanism while avoiding excessive computational complexity. This balance between mathematical accuracy and computational efficiency makes the algorithm particularly suitable for solving extremely large nonlinear systems generated by high-resolution engineering simulations and multidisciplinary optimization problems.

The proposed numerical framework can be readily applied to numerous practical engineering disciplines where nonlinear systems represent the governing mathematical models. Applications include nonlinear structural analysis, electrical power flow studies, computational fluid dynamics, heat transfer simulation, nonlinear optimization, inverse parameter estimation, machine learning model training, robotics, biomedical engineering, environmental modeling, aerospace engineering, and multiphysics simulations. The scalability and computational robustness demonstrated during numerical experimentation indicate that the algorithm is well suited for high-performance scientific computing environments involving millions of nonlinear unknown variables.

Overall, the research establishes that **high-order iterative methods provide a highly effective computational alternative to conventional Newton-type algorithms for solving large-scale nonlinear systems**. The combination of rigorous convergence theory, adaptive numerical implementation, sparse matrix computation, and higher-order correction strategies results in a computational framework capable of delivering rapid convergence, high numerical accuracy, excellent stability, and reduced computational cost. These characteristics

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make the proposed methodology an important contribution to modern computational mathematics and numerical analysis, with considerable potential for addressing increasingly complex nonlinear engineering problems.

Future research may focus on extending the proposed framework toward **Jacobian-free high-order iterative algorithms**, **parallel and distributed implementations for high-performance computing clusters**, **GPU-accelerated nonlinear solvers**, **adaptive preconditioning strategies**, **nonlinear domain decomposition methods**, **automatic differentiation-based Jacobian approximation**, **fractional nonlinear systems**, **stochastic nonlinear equation solvers**, **physics-informed neural network-assisted iterative methods**, **machine learning-based convergence prediction**, **multilevel iterative frameworks**, **nonlinear optimization with constrained systems**, **quantum-inspired numerical algorithms**, and **digital twin-based real-time nonlinear simulation platforms**. These developments are expected to further improve computational scalability, convergence reliability, and practical applicability of high-order iterative methods for solving next-generation large-scale nonlinear engineering and scientific problems.

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